USEFUL FORMULAE

Complex Numbers

general form

$$z = x + iy \qquad \text{or} \qquad z = re^{i\theta} = r(\cos\theta + i\sin\theta)$$

complex conjugation

$$z^* = x - iy \qquad \text{or} \qquad z^* = re^{-i\theta} = r(\cos\theta - i\sin\theta)$$

Determinants

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{aligned}
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} &= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\
&= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} + a_{12}a_{23}a_{31} - a_{12}a_{21}a_{33} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31}
\end{aligned}$$

Integrals

$$\int_{-\infty}^{\infty} \exp(-ax^2)dx = \sqrt{\frac{\pi}{a}}, \qquad (a > 0)$$

$$\int_{-\infty}^{\infty} x^{2n}\exp(-ax^2)dx = 1 \times 3 \times 5 \times \cdots \times (2n-1)\frac{\sqrt{\pi/a}}{(2a)^n}, \qquad (n \geq 1; a > 0)$$

$$\int_0^{\infty} r^n \exp(-ar)dr = \frac{n!}{a^{n+1}}, \qquad (n \geq 0; a > 0)$$

$$\begin{aligned}
\int_0^{\pi/2} \sin^m\theta\cos^n\theta d\theta &= \frac{m-1}{m+n}\int_0^{\pi/2} \sin^{m-2}\theta\cos^n\theta d\theta \\
&= \frac{n-1}{m+n}\int_0^{\pi/2} \sin^m\theta\cos^{n-2}\theta d\theta
\end{aligned}$$

So that

$$\int_0^{\pi/2} \sin^m\theta\cos^n\theta d\theta = \frac{(m-1)(m-3)\cdots(n-1)(n-3)\cdots}{(m+n)(m+n-2)(m+n-4)\cdots} \times C$$

where $C = \pi/2$ if m and n are both even and $C = 1$ otherwise, eg:

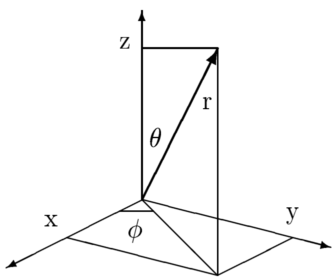
$$\int_0^{\pi/2} \sin\theta\cos^3\theta d\theta = \frac{2}{4.2} = \frac{1}{4}; \qquad \int_0^{\pi/2} \sin^2\theta\cos^2\theta d\theta = \frac{1.1}{4.2}\frac{\pi}{2} = \frac{\pi}{16}$$

Trigonometrical formulae

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\cos A\cos B &= \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)
\end{aligned}$$

$$
\begin{aligned}
\sin A \sin B &= \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right) \\
\sin A \cos B &= \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) \\
\sin A + \sin B &= 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
\sin A - \sin B &= 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\
\cos A + \cos B &= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\
\cos A - \cos B &= -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)
\end{aligned}
$$

Spherical Polar Coordinates



Relationship with Cartesian Coordinates

$$
\begin{array}{ll}
x = r\sin\theta\cos\phi & r = \sqrt{x^2+y^2+z^2} \\
y = r\sin\theta\sin\phi & \theta = \cos^{-1}(z/r) \\
z = r\cos\theta & \phi = \tan^{-1}(y/x)
\end{array}
$$

Integration

$$
\int \cdots d\tau = \int_{r=0}^{\infty}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \cdots r^2 \sin\theta dr d\theta d\phi
$$

Laplacian

$$
\begin{aligned}
\nabla^2\psi &= \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial x^2} \\
&= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}
\end{aligned}
$$