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Symmetry and Symmetry Breaking in Physics: From Geometry to Topology

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Abstract: Symmetry (and group theory) is a fundamental principle of theoretical physics. Finite symmetries, continuous symmetries of compact groups, and infinite-dimensional representations of noncompact Lie groups are at the core of solid physics, particle physics, and quantum physics, respectively. The latter groups now play an important role in many branches of mathematics. In more recent years, we have been faced with the impact of topological quantum field theory (TQFT). Topology and symmetry have deep connections, but topology is inherently broader and more complex. While the presence of symmetry in physical phenomena imposes strong constraints, topology seems to be related to low-energy states and is very likely to provide information about the different dynamical trajectories and patterns that particles can follow. For example, regarding the relationship of topology to low-energy states, Hodge's theory of harmonic forms shows that the zero-energy states (for differential forms) correspond to the cohomology. Regarding the relationship of topology to particle trajectories, a topological knot can be seen as an orbit with complex properties in spacetime. The various deformations or embeddings of the knot, performed in low or high dimensions, allow defining different equivalence classes or topological types, and interestingly, it is possible from these types to study the symmetries associated with the deformations and their changes. More specifically, in the present work, we address two issues: first, that quantum geometry deforms classical geometry, and that this topological deformation may produce physical effects that are specific to the quantum physics scale; and second, that mirror symmetry and the phenomenon of topological change are closely related. This paper was aimed at understanding the conceptual and physical significance of this connection.

Keywords: symmetry; breaking symmetry; geometry; topology; physics; spacetime; deformation; dynamics

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1. Introductory Remarks

"The history of physics is not only a sequence of experimental discoveries and observations, followed by their mathematical description; it is also a history of concepts." W. Heisenberg.

The concept of symmetry has attracted philosophers and mathematicians since the ancient time [1]. The contributions of Plato [2] and Kepler have been extremely fruitful, and their ideas concerning the key function of symmetry in the ordering of the real world are still relevant today. The work of Kepler deserves special comment. He searched for the constitution of physical world. He indubitably believed that the architecture of the cosmos must fulfill certain symmetries. The key to this architecture resided not in numbers but in geometry. Whereas for Galilei, elliptical orbits broke the symmetry of the cosmos [3], for Kepler, the consideration of these orbits paved the way to the discovery of "hidden" deeper symmetries. For him, the veracity of elliptical orbits was a consequence of necessary physical conditions and of laws of harmony [4,5]. With the scientific revolution in the sixth century, the concept of symmetry became the core of a new vision of the universe and of nature [6–9]. This conception strengthened even more with the discovery of relativity theory, both special and general, at the beginning of the twentieth century. However, later on, the fundamental role played by the phenomenon of spontaneous symmetry breaking (SSB) in the organization and evolution of natural and living phenomena was recognized (we refer to [10–17] for a detailed presentation of the subject).

A physical system is *symmetric* if the system remains unchanged when some transformations are applied to it. The key concept behind the fundamental equations of physics is conservation, as defined in the "conservation laws". For an isolated or quasi-isolated system (i.e., in absence of external forces), the most basic physical conserved quantities are energy (which transforms remaining constants), linear momentum (i.e., the product of mass and vector velocity), angular momentum (of an object in circular motion), and electric charge (a very important property of matter when affected by an electromagnetic field). It must be specified, however, that the momentum for a massless particle, like photon, is not the product of the mass and the velocity (which is 0). This is quite relevant for the theory of special relativity, wherein the concept of relative mass (defined by Einstein in 1905) and the related mass–energy equivalence explain why gravity may have an influence on light (photons). Special relativity proved that a photon does have relativistic mass proportional to its momentum. Furthermore, De Broglie's relation, concerning quantum theoretical wave–particle duality, yields (assuming that m is relativistic mass) the important result that photons have mass inversely proportional to their wavelength.

The *symmetry principle* plays a fundamental role in mathematics and in the natural sciences, particularly in physics. According to this principle, the group of symmetries governing the causes of a physical system form a subgroup belonging to the group of symmetries resulting from the effects. Differently stated, the effects can have a larger group of symmetries than the causes. However, these principles, in many situations, are contradicted by the phenomenon of "spontaneous symmetry breaking", which means that there are physical systems where the effects simply have less symmetry than the cause. In other words, the actual physical world is less symmetric than the fundamental equations of physics. Mathematically, this can be expressed by saying that the symmetries that these equations obey are more general than the symmetries of the physical solutions of the equations when spontaneous symmetry breaking occurs. This claim should, however, be completed by saying that the spontaneous breaking symmetry mechanism is often strictly connected with the dynamical rearrangement of symmetry. For example, in quantum field theory, the symmetry at level of asymptotic fields is the group contraction of the symmetry at level of interacting fields (same number of generators). For more details, see [18–21].

Spontaneous symmetry breaking occurs in superconductivity, the electroweak interaction, early universe formation (as predicted by inflationary cosmology), and many other physical contexts. Well-known examples of this mechanism of breaking-symmetry are the phase transition from a liquid to a crystal and the violation by the "standard model" of the symmetry between matter and antimatter.

2. The Groups of Minkowskian and Einsteinian Spacetimes

Let us now present the most important conceptions of space–time by comparing them with respect to their respective groups of symmetries. The relation between Minkowskian and Einsteinian spacetimes mirrors that between Galilean and Newtonian spacetimes (see [22–24]). Thus, Minkowskian spacetime has a unique description (for example, its geodesic structure is the same as that of R^4, the set of time-like geodesics corresponding to geodesics in R^4 making an angle less than 45° with some fixed direction), and it does not describe gravitation. It has a ten-parameter transitive symmetry group of motion, namely, the Poincaré (i.e., the inhomogeneous Lorentz) group. According to Stachel [25], "Any set of dynamical variables must form a representation of this group and obey dynamical equations that transform appropriately under it. More precisely, the set of dynamical variables at each point must form a representation space of the homogeneous Lorentz group. For fields, the dynamical equations must guarantee that these representations together form a representation of the inhomogeneous Lorentz (Poincaré) group. Theories